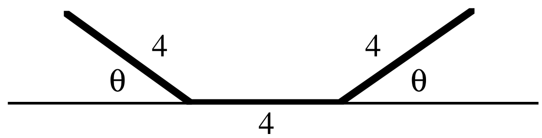
Modeling With Right Triangle Trigonometry

1. **The Gutter Problem:** A rain gutter is to be constructed by taking an aluminum sheet 12 inches wide, marking off 4 inches from each edge, and bending up the side at an angle θ .



Question: What angle should we make θ , in order to maximize the cross sectional area of the gutter? (This bend will allow the most water to flow through the gutter.)

What is the cross sectional area A of the opening of the rain gutter if the angle θ is 25°?
Hint: Divide the cross sectional area into sections and find the missing (and important) parts of each section.

Area = ________________

Write a function $A(\theta)$ for the cross sectional area of the opening of the rain gutter for any angle θ . Use algebra to simplify the expression.

$A(\theta)$ = ____

= ____

With your calculator, sketch a graph of the area function $A(\theta)$. (Again, think about DEGREES vs. RADIANS.) Think about the values of θ that make sense to the problem.

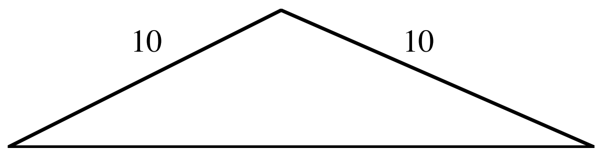
Using your graph, find the angle θ that maximizes the area of the opening. What is the maximum cross sectional area?

The maximum area occurs when θ is ________. The maximum area is ___________________ in^2.

Using your graph, find the angle θ that would cause the area of the opening to be 15 in^2.

θ = ________________

2. Isosceles Triangle Problem: The two equal sides of an **isosceles triangle** have a length of 10 cm, as shown below.



a. Let the <u>base angles</u> of the triangle measure 20 degrees.

i. Calculate the **perimeter** of the triangle.

ii. Calculate the **area** of the triangle.

b. If the <u>base angles</u> of the triangle measure θ degrees, determine a function $P(\theta)$ for the **perimeter** of the isosceles triangle, and a function $A(\theta)$ for the **area** of the isosceles triangle.

$P(\theta) =$ ______________________________ $A(\theta) =$ ______________________________

c. With your calculator, graph the function $P(\theta)$, and use your graph to determine the angle θ that will make the perimeter 25 cm.

$\theta =$ ____________

d. With your calculator, graph the function $A(\theta)$, and use your graph to determine the angle θ that will

i. make the area of the triangle 30 cm^2.

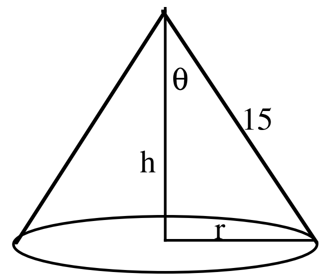
$\theta =$ _____________

ii. maximize the area of the triangle.

$\theta =$ ______________

3. **The Cone Problem**: Shown to the right is a cone with a slant height of 15 cm. Let's explore the relationship between the volume of the cone and the angle θ .

Remember, for a cone, the Volume = $\frac{1}{3} \cdot \pi \cdot \text{radius}^2 \cdot \text{height}$.



Question: What angle should we make θ in order to maximize the volume of the cone.?

Let's take a small angle for θ , for example **5 degrees**. Since the volume of the cone is dependent on the radius and the height of the cone, we need to calculate the radius and the height if the angle θ is 5 degrees.

Write a trig equation that you could use to determine the **radius** of the cone. Solve the equation to find the **radius**.

If θ = 5 degrees, **r =** ________________.

Write a trig equation that you could use to find the **height** of the cone. Solve the equation to find the **height**.

If θ = 5 degrees, **h =** ______________

What is the **volume** of the cone if the angle θ is 5 degrees?

Volume = ____

To determine the "exact" value of θ that **maximizes the volume of the cone,** we need to write a function for the **volume** in terms of the angle θ . Use trig to write the **radius** and the **height** of the cone in terms of θ .

radius = _________________________ **height =** _________________________

Using these two equations, write an equation for the **volume** of the cone in terms of θ .

Volume = ____

Graph this function on your calculator. Think about the window you need!

Now, use your grapher to complete the following:

1. The **maximum volume** for a cone with slant height of 15 cm is ________________ and it occurs when the:

   angle θ is ______________, the **radius** is ________________, and the **height** is. _________________.

2. If the volume of the cone is 300 cm^3, what would the angle θ be? ____________________